Math 1
U3 L3 System Word Problems

Name: KEY 2/5/13

For each problem, identify your variables (define), set up a system of equations (2 equations, 2 variables), and then solve the system. State your answer in terms of the problem.

Example:

The sum of two numbers is 92. Their difference is 20. Find the two numbers.

x = first number

y = second number

$$\begin{cases} x+y=92 \\ x-y=20 \end{cases} \rightarrow x = 92 - y$$

$$x = 92 - 36$$

$$\boxed{x = 56}$$

$$(92 - y) - y = 20$$
$$92 - 2y = 20$$
$$-2y = -72$$
$$\boxed{y = 36}$$

1. The difference of two numbers is 16. The greater number is 5 less than 4 times the smaller number. Find the two numbers.

x = first number (larger)
y = second number (smaller)

$$\begin{cases} x - y = 16 \\ x = 4y - 5 \end{cases}$$

$$x = 4(7) - 5$$
$$x = 28 - 5$$
$$\boxed{x = 23}$$

$$(4y - 15) - y = 16$$
$$3y - 15 = 16$$
$$3y = 21$$
$$\boxed{y = 7}$$

2. A 100-foot cable is cut into two pieces. The first piece is 18 feet longer than the second. How long is each piece of cable?

x = first piece
y = second piece

$$\begin{cases} 100 = x + y \\ x = y + 18 \end{cases}$$

$$x = 41 + 18$$
$$x = 59$$

$$100 = (y + 18) + y$$
$$100 = 2y + 18$$
$$82 = 2y$$
$$41 = y$$

First piece is 59 ft. and second piece is 41 ft.

3.   Three apples and four bananas cost \$4.85. Three apples and ten bananas cost \$8.75. Find the cost of an apple.

Above the problem: $\overset{a}{\text{apples}}$ and $\overset{b}{\text{bananas}}$

$$-1\begin{cases} 3a+4b=4.85 \\ 3a+10b=8.75 \end{cases} \rightarrow \begin{cases} -3a-4b=-4.85 \\ +\; 3a+10b=8.75 \end{cases}$$

$$6b=3.9$$
$$b=0.65$$

$$3a+4(0.65)=4.85$$
$$3a+2.6=4.85$$
$$3a=2.25$$
$$a=0.75$$

$$\boxed{\begin{array}{l} \text{apples}=75\cent \\ \text{bananas}=65\cent \end{array}}$$

4.   The Rocket Coaster has 15 cars, some that hold 4 people and some that hold 6 people. There is room for 72 people altogether. How many 4-passenger cars are there? How many 6-passenger cars are there?

$$f = \text{four-passenger car}$$
$$s = \text{six-passenger car}$$

$$\begin{cases} 4f+6s=72 \\ -4(f+s=15) \end{cases} \rightarrow \begin{cases} 4f+6s=72 \\ -4f-4s=-60 \end{cases}$$

$$2s=12$$
$$s=6$$

$$f+6=15$$
$$f=9$$

$$\boxed{\begin{array}{l} 9 \text{ four-passenger cars} \\ 6 \text{ six-passenger cars} \end{array}}$$

5.   Tickets for the Valentine Dance cost \$3 per person or \$5 per couple. If \$475 worth of tickets were sold and 180 people attended the dance, how many couples were there?

$$p = \text{person}$$
$$c = \text{couple}$$

$$\begin{cases} 3p+5c=475 \\ p+2c=180 \end{cases}$$

$$p=180-2c$$

$$3(180-2c)+5c=475$$
$$540-6c+5c=475$$
$$540-c=475$$
$$-c=-65$$
$$c=65$$

$$p=180-2(65)$$
$$p=50$$

$$\boxed{\begin{array}{l} 50 \text{ single people} \\ 65 \text{ couples} \end{array}}$$

6. Pi High School ordered 40 science books. The next week, the school ordered 30 algebra books. The bill for the first order was $360 greater than the bill for the second order. The two bills together totaled $3960. Find the price of an algebra book.

$$\begin{cases} 40S + 30a = 3960 \\ 40S = 30a + 360 \end{cases}$$

$$S = \frac{3}{4}a + 9$$

$$40\left(\frac{3}{4}a + 9\right) + 30a = 3960$$
$$30a + 360 + 30a = 3960$$
$$360 + 60a = 3960$$
$$60a = 3600$$
$$a = 60$$

$$\boxed{\text{Algebra price} = \$60}$$

7. Dante has 27 coins that are all dimes and quarters. The value of the coins is $4.35. How many dimes and how many quarters does Dante have?

$$q = \text{quarter}$$
$$d = \text{dime}$$

$$q = 27 - d$$

$$\begin{cases} q + d = 27 \\ 4.35 = 0.25q + 0.10d \end{cases}$$

$$q = 27 - 16$$
$$q = 11$$

$$4.35 = 0.25(27 - d) + 0.10d$$
$$4.35 = 6.75 - 0.25d + 0.10d$$
$$4.35 = 6.75 - 0.15d$$
$$-2.40 = -0.15d$$
$$16 = d$$

$$\boxed{\begin{array}{c} 11 \text{ quarters} \\ \text{and} \\ 16 \text{ dimes} \end{array}}$$

8. The larger of two numbers is 11 more than twice the smaller number. The sum of the numbers is 1 less than seven times the smaller number. Find the numbers.

$$x = \text{larger}$$
$$y = \text{smaller}$$

$$\begin{cases} x = 2y + 11 \\ x = 6y - 1 \end{cases}$$

$$x + y = 7y - 1$$
$$x = 6y - 1$$

$$2y + 11 = 6y - 1$$
$$-4y + 11 = 1$$
$$-4y = -12$$
$$y = 3$$

$$x = 2(3) + 11$$
$$x = 6 + 11$$
$$x = 17$$

$$\boxed{\begin{array}{c} \text{Numbers are} \\ 17 \text{ and } 3 \end{array}}$$

9. A geometry teacher has a set of 60 plastic pentagons and octagons. She happened to notice that all the figures together have a total of 354 sides (she had lots of spare time to count). How many of each shape are there?

P C

$$\begin{cases} P + C = 60 \\ 5p + 8C = 354 \end{cases} \longrightarrow p = 60 - C$$

$5(60 - C) + 8C = 354$ $p = 60 - 18$

$300 - 5C + 8C = 354$ $p = 42$

$300 + 3C = 354$

$3C = 54$

$C = 18$

| 42 pentagons and 18 octagons |

10. A group of Mayfield students goes out to lunch. If two have burritos and five have tacos, the bill will be \$19.50. If five have burritos and two have tacos, the bill will be \$22.50. Find the price of a taco and the price of a burrito.

$t = taco$

$b = burrito$

| Taco is \$2.50 and burrito is \$3.50 |

$-5(2b + 5t = 19.50)$ $\begin{cases} -10b - 25t = -97.50 \\ +10b + 4t = 45 \end{cases}$

$2(5b + 2t = 22.50)$

$\hspace{6cm} -21t = -52.50$

$2b + 5(2.5) = 19.50$ $t = 2.5$

$2b + 12.5 = 19.50$

$2b = 7$

$b = 3.5$

11. Three lemon cookies plus two fudge cookies have 400 calories. Two lemon cookies plus three fudge cookies have 425 calories. How many calories are in each kind of cookie?

$L = lemon$

$f = fudge$

$-2(3L + 2f = 400)$ $\begin{cases} -6L - 4f = -800 \\ +6L + 9f = 1275 \end{cases}$

$3(2L + 3f = 425)$

$\hspace{6cm} 5f = 475$

$3L + 2(95) = 400$ $f = 95$

$3L + 190 = 400$

$3L = 210$

$L = 70$

| Fudge cookies are 95 calories and lemon cookies are 70 calories |

12. In Lewis Carroll's *Through the Looking Glass*, Tweedledum says, "The sum of your weight and twice mine is 361 pounds." Then Tweedledee says, "The sum of your weight and twice mine is 362 pounds." Find the weights of Tweedledum and Tweedledee.

$x = $ dum
$y = $ dee

Tweedledum is 120 lbs.
Tweedledee is 121 lbs.

$$\begin{cases} y + 2x = 361 \\ x + 2y = 362 \longrightarrow x = 362 - 2y \end{cases}$$

$$y + 2(362 - 2y) = 361$$
$$y + 724 - 4y = 361$$
$$724 - 3y = 361$$
$$-3y = -363$$
$$y = 121$$

$$121 + 2x = 361$$
$$2x = 240$$
$$x = 120$$

13. The difference of two numbers is 15. Five times the smaller number is the same as 9 less than twice the larger number. Find the numbers.

$x = $ first number (larger)
$y = $ second number (smaller)

$$\begin{cases} x - y = 15 \longrightarrow x = 15 + y \\ 5y = 2x - 9 \end{cases}$$

$$5y = 2(15 + y) - 9$$
$$5y = 30 + 2y - 9$$
$$5y = 21 + 2y$$
$$3y = 21$$
$$\boxed{y = 7}$$

$$x - 7 = 15$$
$$\boxed{x = 22}$$